iMuSciCA Scenario for Lower Secondary Education

Scenario 4: Create a piece of music using geometric symmetries

Title:	1st degree Secondary education Scenario 4: **Create a piece of music using geometric symmetries**		
Keywords:	Music: motif, rondo, ostinato, sequence, augmentatio, diminutio Mathematics: symmetry, translation, reflection, rotation, enlargement		
Short description:	iMuSciCA Scenario on transformations in mathematics and patterns in music. Depending on the lesson hours and on the subject (mathematics-physics-music-STEM) this scenario is performed in class, the teacher may place different emphasis in the different parts of the scenario dealing with those specific subjects.		
Accompanying Lesson plans:	Lesson plan 4.1: **Discovering symmetry in music and mathematics** Lesson plan 4.2: **Transformations in music and mathematics** Lesson plan 4.3: **Combinations of transformations in music and mathematics**	Date:	07/05/2018
Educational Goals:	- Recognition of musical and mathematical patterns - Associating a mathematical translation with a musical ostinato - To study a reflection in mathematics as well as in music - Associating a mathematical rotation with similar motif processing as music - Associating mathematical enlargements with an augmentation ratio and diminutio from the world of music	Estimated length:	4-6 h A minimum program can be Lesson 1, 2

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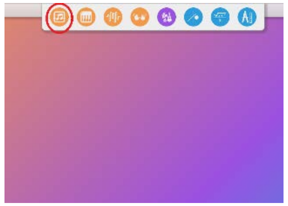
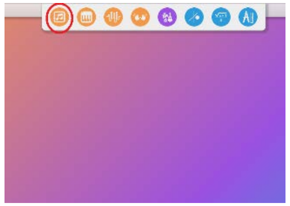
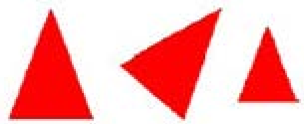
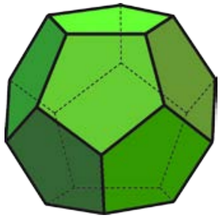
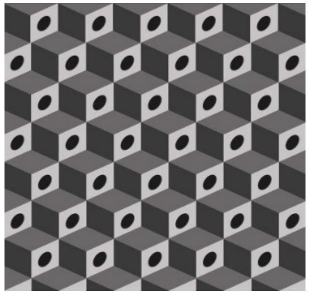
Collaborator(s):	Erica Andreotti, Renaat Frans, Jeroen Op den Kelder, Janne Colla, Arne Duyver	Language:	Dutch
Status:		Difficulty degree:	Low
Dissemination level:		Special Needs Addressed:	

Lesson plan 4.1: Discovering symmetry in music and mathematics - In detail

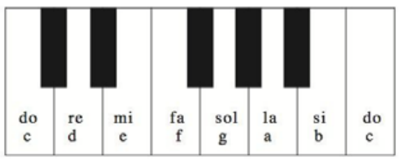
Time	Phases	Field	Description	Activity	Remarks
	Engage/ Imagine	A	We start in the musical world and study patterns in music.	**Patterns in music:** **Research question: How do you create music? Are musical notes simply randomly placed one after the other on a score or do musical notes in a certain piece do occur in a certain pattern, in a certain structure?** Watch and listen to the movie below. https://www.youtube.com/watch?v=MAY1UoQYMHk&index=4&list=PL2JLK93IjQUKKBxpkXNGJwEbUbwr93Yis  Do you recognize patterns in this piece of music? (Yes) Do you hear pieces of music coming back? (Yes) Do you see certain drawings returning? (Yes) Open the DrawMe tool in the workbench and follow the instructions below. Listen to this piece of music by pressing the play button. 	Use the iMuSciCA workbench DrawMe-tool. 

				The smallest pieces of a piece of music that serve as a building block for a composition are called **motifs**. In the DrawMe drawing, the green drawning for instance represents a motif. A motif is a small piece of music that a composer repeatedly makes small variations of in order to arrive at a beautiful composition. A composition normally consists of several musical motifs.	
				Conclusion Tick the appropriate box: o In a music composition we can recognize patterns. o In a music composition we cannot recognize patterns.	
	Analyse Communicate Reflect	S	Now we are going to investigate whether specific patterns in mathematics are also recognizable.	**Research on patterns and symmetries in mathematics.** **Research question: Music pieces are full of symmetries and patterns. The same goes for mathematical figures. Let us discover symmetries and patterns in art and maths.** Look at the triangles below. If you look at the 2^{nd} and the 3^{rd} triangle they change something comparted to the 1^{st}, but they keep also something. Changing and keeping something, that's what is called a symmetry? What symmetry can you discover between the different triangles? (2^{nd} triangle rotation of the 1^{st}, 3^{rd} triangle rotation and reduction compared to the first)  See the figures and expressions below:	Certain patterns and symmetries are also present in mathematics.

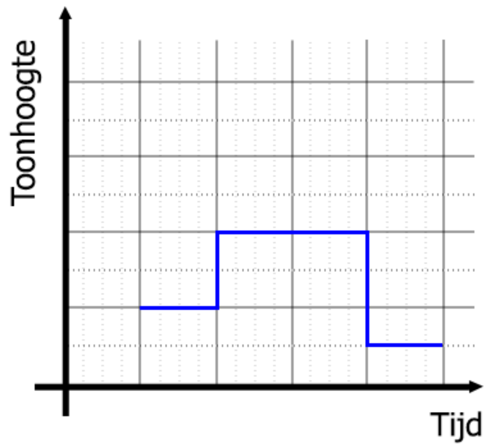
				Can you find the flat figure from which the geometric shape (Dodecahedron) is built? (Yes, a regular pentagon).  Can you discover a pattern in each of the two expressions below? ∞ (reflection through a vertical line in the middle, or rotation over 180° for that matter. Also a reflection through a horizontal line in the middle $\frac{2}{5}$ (reflection of number 2 with respect to the break line to number 5).	
				Look at the drawing below based on the work of the artist Maurits Cornelis Escher. Can you find the figure with which the whole work is built? (Yes - the block) This can be compared to a motif in a piece of music. Can you discover the **pattern** in the drawing? (Yes - the block is translated each time). You can compared this repetition to a certain motif in music that a musician repeats many times in order to create a completer piece of music.	

					
	Analyse			**Conclusion:** Tick the appropriate box: o In mathematical figures we can recognize patterns and symmetries. o In mathematics figures we cannot recognize patterns and symmetries	
				Conclusion: Patterns are not strange to us, unconsciously we look for symmetry and we recognize this in art, in mathematics, in nature ... and also in music. By using symmetry in music we obtain unity and there is a connection between the different musical parts. Motifs in music are in general not random but are connected to each other by symmetry operations. That's why it's interesting to explore symmetry **in both mathematics and music**. Moreover, a music composer who incorporates symmetries into his composition will not only create **unity in the multiplicity** but will also do a lot **with few elements/ building blocks.**	
	Communicating/ Reflecting	S	What have you discovered in this lesson plan? Discuss and come to conclusions.	Show what you have discovered so far. Do this by working with the teacher and all other students to design a diagram that summarizes the lesson in a structured way.	
				We have discovered the existence of patterns and symmetries, both in the world of music and in mathematics! We are now going to look into this further.	

Lesson plan 4.2: **Transformations in music and mathematics**

Time	Phases	Field	Description	Activity	Remarks
	Engage /Imagine	A	We start in the musical world and look at motif processing in music. To do this, we first develop a more visual notation of tones.	**Research question: What patterns, symmetries does a musician use to write a piece of music?** Before the research question is addressed, we develop a visual notation of musical tones, a notation that everyone can read, especially those who can't read notes on a score. **First step: the musical interval** We explain the principle of full and half tones using a keyboard. **Assignment**: Look at the keyboard and accompanying music score below. The principle of a keyboard is that there is **one semitone between each key and the next** (the black keys also count).  What is the musical interval, expressed in semitones between: - do (C) and re (D)? (Answer: 2 semitones or one whole tone) - mi(E) and fa(F) (Answer: one semitone) **Second step: the duration of the tone** A tone can last *two* counts or *one* count, one *half* count or one *fourth* count. Look at a normal score below to see how these notes are written down.	In order to let students experience and understand this scenario with minimal knowledge of music, a more visual representation of music notation is made based on line constructions.

Assignment:
Check out the score below. Find the distance between the notes and view the duration of the tones. Look at the line construction below (stepwise).
Do you understand the connection with the score in the example?



Solution:
The first blue horizontal line represents the notation of the F (fa), the second horizontal line of the staircase represents the 2-counting G (sol). This G is twice as long. The last horizontal line represents the E (mi) and is again 1 count long..

Assignment :

<table>
<tr>
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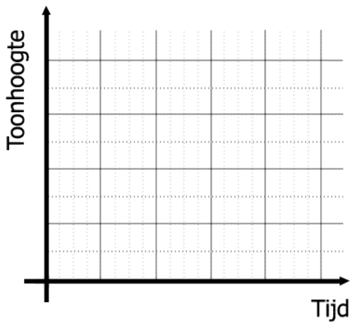
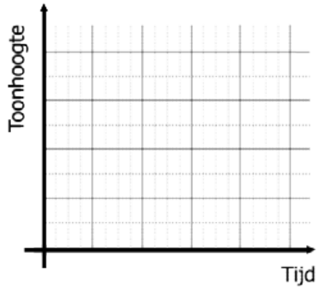
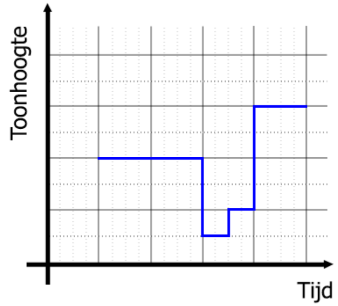
<td>In the empty grid, draw the lines belonging to the score below and compare your structure with that of your fellow students. Look back to the keyboard to see how many (semi-)tones are between the notes.

Assignment 2:
Draw the lines in the empty grid that belong to the note score below and compare your structure with that of your fellow students.


</td>
<td>Sollution:

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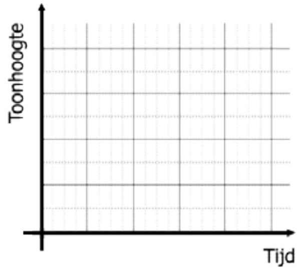
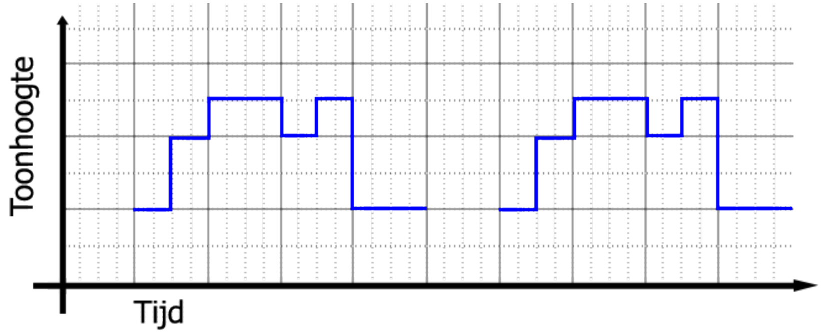
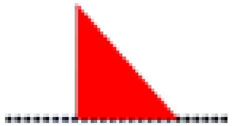
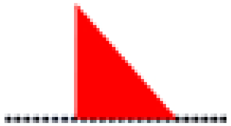
				Assignment 3: Draw the lines in the empty grid that belong to the note score below and compare your structure with that of your fellow students.   	
	Engage /Imagine	A	We stay in the musical world and look at the specific motif processing: *the translation*. We discuss translations in different dimensions: translation in time, translation in pitch and a combination of both.	We return to our research question: "**What patterns and symmetries does a musician use to compose a piece of music?"** A first symmetry operation (motif processing) is the repetition or also called a *translation in time*. **A repetition in music (or: a translation in time)** **Research question: A musician regularly uses repetition as the simplest way of processing motifs.** **How can we recognize a translation in time on a score? How can we note a translation in our visual representation?** **Assignment**: Listen/see the piece of music below and watch the score carefully. The line construction of the score is also shown as an extra tool. You have a lot of 'cards' and on each of these cards there is a line construction representing a musical motif. Find the right line construction that matches the motif.	Translations occur regularly in music. To better understand this phenomenon we will dive into mathematics where the concept of translation also exists. In order to make the connection between the two worlds clear, we use a schematic and visual representation of the score (based on line constructions). We will reason at the same time in the mathematical world on a flat figure like a triangle. In this way we let you discover the connection between mathematical and musical symmetry.

Musical piece:





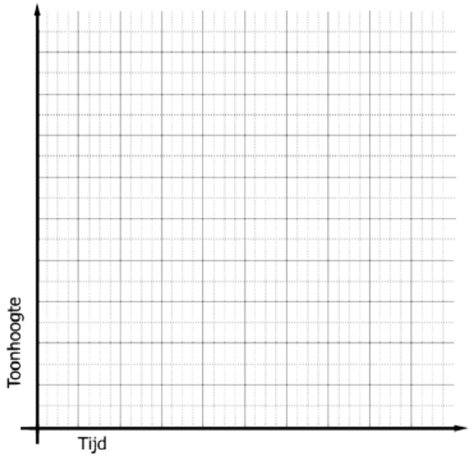


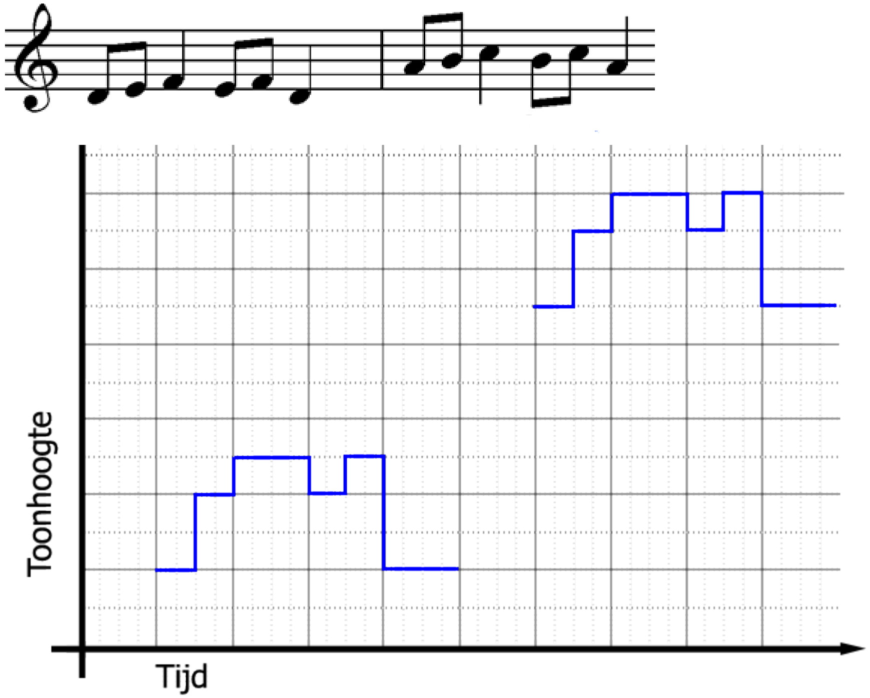
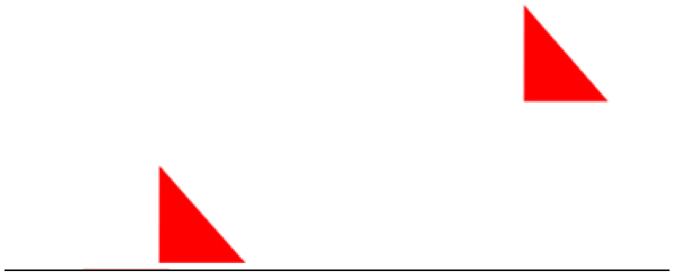
Solution:
The first piece of music (score part 1) is the same as the second piece of music (score part 2) only translated in time.
The notes are translated to the right, as it were.

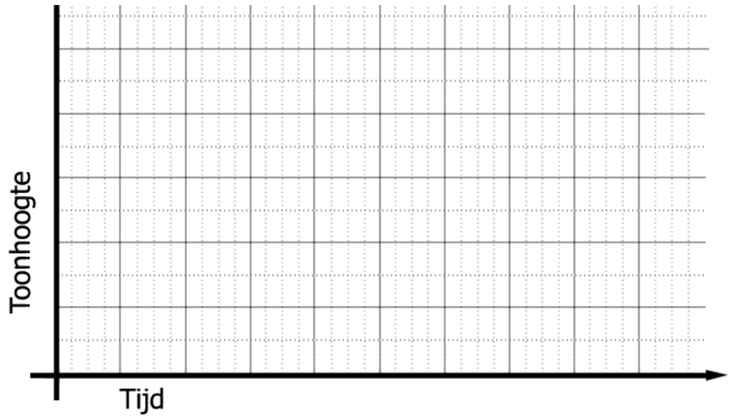
By 'the first piece of music' we mean a certain motif. A motif is a building block of a piece of music that consists of a number of notes. The second piece of music' is another motif. We obtain the second motif by translating the first motif in time. Starting from a motif, you can create an infinite number of other motifs by transforming them.

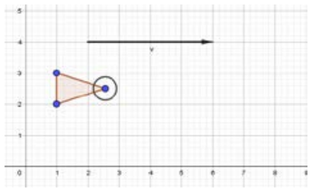
There are cards going with this lesson plan. On each of these cards there is a line construction representing a musical motif. When you print these cards out your students can work with these cards to solve the different assignments in combination with iMuSciCA's workbench tools DrAwMe and iMuSciCA's Geometry and Algebra tools.
You find a printable file:

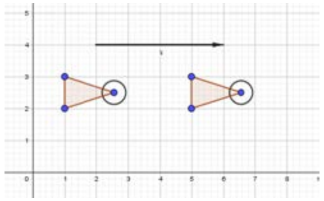
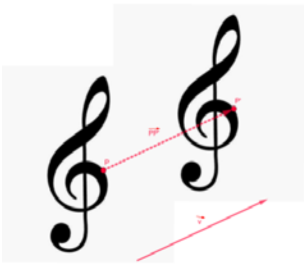
- With the cards
- To make a transparent of one of the figures

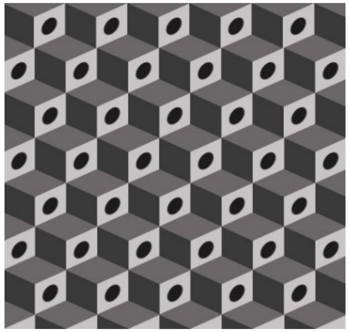
				In turn, translations do exist as well in the mathematical world. Let us show this with with a geometrical figure like the triangle. It is clear that the second triangle is obtained by moving the first triangle horizontally. This way you can visually perceive the translation in time even more easily without reading the notes.. **Assignment**: Discover the repetition in the following piece of music, note down the lines in the empty grid.   	
				A translation in pitch (sequence) **Assignment:** Watch the piece of music below, do you notice the processing of motifs? It is again some kind of translation but no the translation is not in time but in …..? (This is a translation in pitch, which is called a sequence in music).	

				 	**Solution:** The first piece of music (score part 1) is the same as the second piece of music (score part 2) only at a different pitch. The notes have translated upwards, as it were.

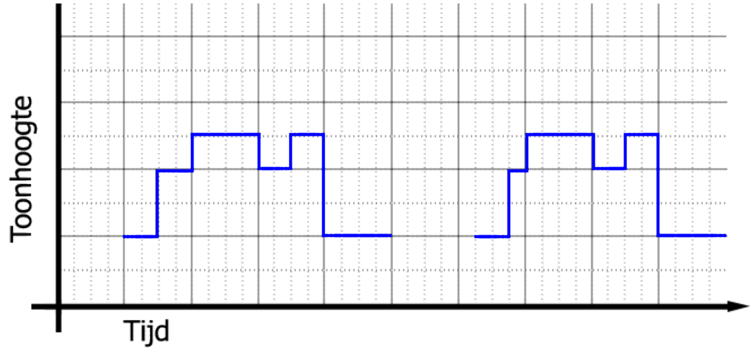
				Assignment: Discover the translation in pitch in this piece of music below, note down the lines in the empty grid.   **Conclusion:** In music one can consider a translation in two ways: a translation in time, but also a translation in pitch. We call a translation in time a **repetition**. A translation in pitch is called a **sequence**. You can also keep repeating an entire piece of music for a certain motif. If that is the case, then we speak of an **ostinato**, named after ‘ostiner’, which means 'persevering'. The same piece of music is thus repeated throughout the whole piece of music or song, or translated in time.	

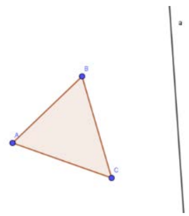
	Engage /Imagine	S	Carrying out a translation in mathematics first with concrete material	**A translation in mathematics** **Research question: What is a translation in mathematics? How can we describe a translation mathematically?** **Assignment**: Place a figure, which you cut out of cardboard, on your table and move the figure over a distance of 4 cm, horizontally, to the right. The object can be moved to the left or to the right, to the bottom or to the top. The orientation that is being translated to is therefore important. Mathematically, we say that the figure is translated over a **oriented line segment or vector**. A oriented line segment has a length, a direction (vertical, horizontal) and a sentence (left or right, up or down). In this assignment the line segment has a length of 4 cm, horizontal translation with the direction to the right.	Definition of an oriented line segment.
	Investigate / Analyse	S	Performing a translation in mathematics with the Geometry and Algebra tool on the iMuSciCA	**A translation in mathematics** **Assignment**: Draw an object of your choice in the Geometry and Algebra tool on the iMuSciCA workbench (below a triangle is drawn as an example) and move the object over a vector of any length, horizontally, pointing to the right. **Solution**: 	Use the Geometry and Algebra tool on the iMuSciCA workbench: the far right button.  Have a look at the help document and discover how geometical symmetry operations can be performed with iMuSciCA's Geometry & Algebra tools.

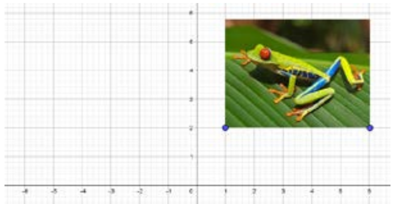
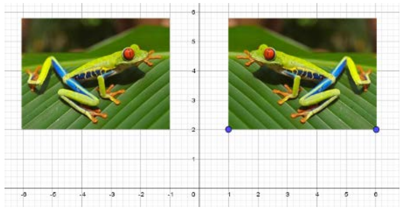
					
			The mathematical formulation of a translation	**A translation in mathematics** **Assignment:** Look at the figure below. Do you see the translation of the treble clef (G-clef) over a vector v? Any point P of the image of the original treble clef is translated to a point P' of the translated image of the treble clef.  **Comment**:	We give here a practical example of a mathematical translation without presenting the complete mathematical definition. You can focus on this more during maths class. Notice the difference between translations in mathematics and in music: In music you can move in 2 different dimensions: in pitch and in time (or even a combination of those 2). In geometry on the other hand you in principle consider is a translation in a space dimension.

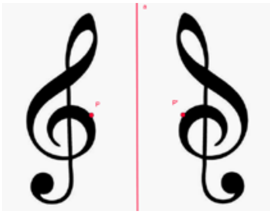
				In mathematics, the oriented segment/vector can be horizontal, vertical or oblique.	
	Engage	S	Applying the mathematical translation	**A translation in the visual arts** **Assignment:** • Take a look at the figure below based on the work of the artist Maurits Cornelis Escher. What transformation did the artist Maurits Cornelis Escher use to make his works of art? (- translation)  • Draw the oriented segment that shows/slides a cube directly onto the cube in front of it. • Imagine that the figure continues indefinitely. If we move each cube one place forward, will we get the same figure? (-Yes) **Conclusion:** An object depicted by a translation in itself is called a **translational symmetry**.	The concept of *translational symmetry* is being addressed here. TIP: You can make the connection between an ostinate in music and the concept of translational symmetry in mathematics.

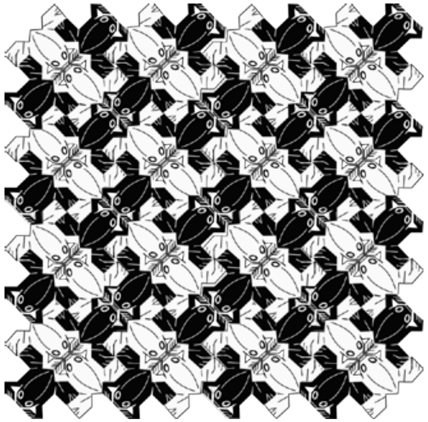
	Create	A	Create your own piece of music with a translation in it as motif processing.	**Assignment:** Create a piece of music yourself in the DrawMe tool in which a self-designed motif is translated. **Possible solution:**  Here a section of the pink motif (do - re - mi - do) is repeated.	Use the DrAwMe tool in the iMuSciCA workbench https://workbench.imuscica.eu/
	Engage /Imagine	A	We stay in the musical world and look at the motif processing: the varied repetition (This section is **optional** in the scenario)	**Assignment**: Watch the piece of music below. Each time, you 'll find the schematic representations of the score: both the visual line construction and also the triangle. Can you find a repeated/translated motif somewhere? 	In music one speaks of a *varied repetition*. In mathematics, as an exact science, such an imperfect translation is not seen as a translation though it almost is one. In the music these 'imperfect translations' appear, just musically very interesting! This assignment **can possibly** be **skipped** and you can directly go on with the reflection if you want. This does not make continuing the rest of the scenario impossible.

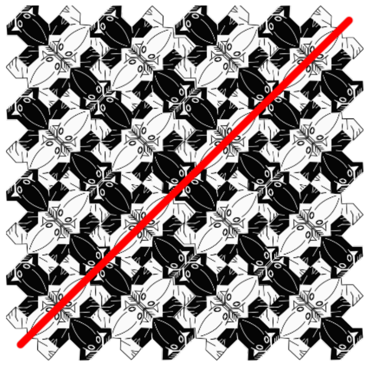
				   **Solution**: If your answer to the above question is no, then you think very mathematically (which is certainly not wrong). It is a repetition, but with a minor change. In music this is called a **varied repetition**. In mathematics, however, we do not call it a repetition or a translation, because the sliding-screen is not the same as the original, though it is *almost* the case. In music or other art forms, people do like this 'small lack of symmetry'. A small deviation from perfection is considered beautiful, think for example of the shape of your own face. Now that you know this, can you find the varied repetition in the above piece of music?	
	Investigate / Analyse	S	What is the relationship between the varied repetition in music and the mathematically exact translation?	**Research question: What is the connection between the musical varied repetition and the mathematically exact translation?** **Assignment**: Examine following piece of music below. Find the mathematical abnormality in this musical varied repetition.	

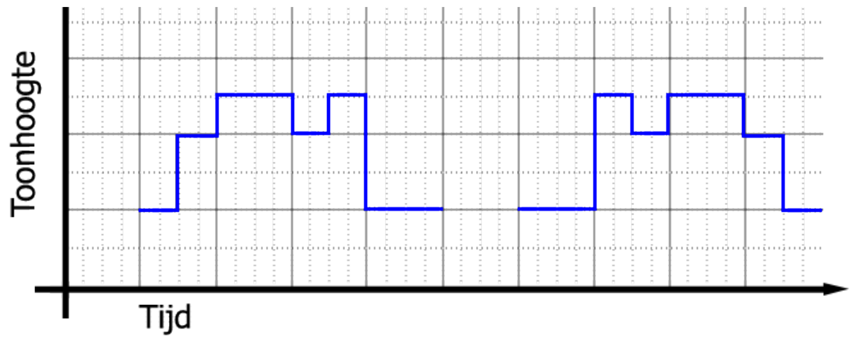
				(Hint: Which motif is 'translated' here, what is the mathematical difference in the sliding-screen?) 	
	Engage /Imagine	S	First, a reflection in mathematics should be carried out by students using concrete materials.	**A reflection in mathematics** In order to be able to answer the research question "A musician regularly uses a reflection in his piece of music" first we dive into the world of mathematics. **Research question: What is a reflection in mathematics? How can we describe a reflection mathematically?** **Assignment**: • Take the small sheet of paper with triangle ABC and a straight line a printed on it. Have a needle, pricking mat and mirror at hand. Place the mirror on the straight a with the mirror side facing the triangle. What do you see in the mirror? (- the mirror image of triangle ABC.)  • How do you get this virtual mirror image on your sheet of paper? Fold the sheet of paper in half on line segment a and prick out the triangle on the pricking mat. Fold the sheet of paper back open. What can you see on your sheet of paper? (- that same mirror image.)	Reflections occur regularly in music. To better understand this phenomenon, we first dive into mathematics where the concept of reflection also exists. **Concrete didactic materials needed:** puncture needle, pricking cushion, pricking drawing, mirror The definition of a mirror axis is made.

				• Measure the perpendicular distance from 'corresponding' points to the straight line a (e.g. perpendicular distance between point A and its mirror point A). What can you decide about the straight a? • (- Straight a is the perpendicular line between the original points and their mirror images). **Definition**: The straight line around which we reflect/mirror the image is called the **mirror axis**. In the command straight a is the axis of symmetry.	
	Investigate / Analyse	S	Performing a reflection in mathematics using iMuSciCA's Geomtery and Algebra tools.	**A reflection in mathematics** **Assignment**: Draw an object of your choice or insert an object of your choice into iMuSciCA's Geometry and Algebra tool and mirror the object around the y-axis. Compare your result with the drawings below:  	Use iMuSciCA's Geometry and Algebra tools: the far right button.  Have a look at the help document and discover [how geometrical symmetry operations can be performed with iMuSciCA's Geometry & Algebra tools].
			The mathematical formulation of a reflection.	**A reflection in mathematics** **Assignment:** Look at the figure below. Do you see the reflection of the treble clef around the mirror axis a? Any point P of the original clef is mirrored around the axis a in a point P' of the mirror image of the clef.	After pupils have first done a reflection with concrete material and afterwards with iMuSciCA's software, they are ready to understand the mathematical meaning of reflection.

				 **Remark**: Notice that the straight line a is the perpendicular line between each point P and its mirror image P'. **Assignment:** Take a look at the pictures below. Can you find the mirror axis? Draw the mirror axis on each photo.  	Here we give again the mathematical essence without, however, giving an exact definition. You can focus on this further in a maths class.

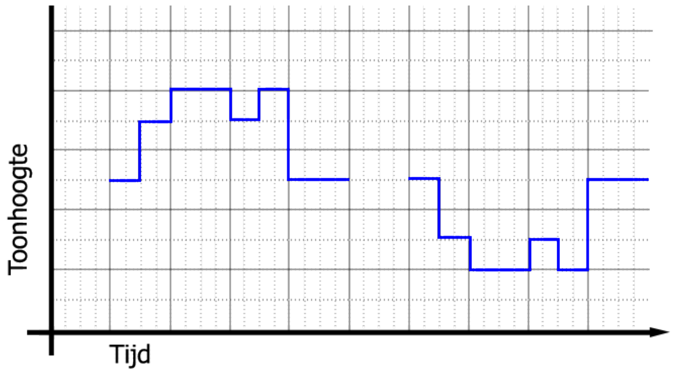
	Apply	S	Applying the definition of reflection	**Assignment:** Look at the figure below. What transformations could the artist have used to make the work of art? (- Reflection or translation) Verify your answer by drawing on the figure.  **Solution**: Draw a mirror axis that causes a white frog to be displayed on the white frog opposite to him, to show how the reflection works. Draw an oriented segment that images a white frog on the white frog to the right of him. This oriented segment line shows then clearly the translation that is used in this piece of art.	
				Assignment: Imagine that the figure below runs infinitely in all directions. When you place a mirror axis between e.g. 2 black frogs (as shown below) you still get the same figure? (- yes)	The term axial symmetry is being addressed here.

				 **Conclusion**: A figure that is imaged by a reflection on itself is called line symmetrical.	
	Apply	M	We return to the music. What is learned about reflection from mathematics is applied. In musical pieces, mirroring is only done around horizontal or vertical lines, no oblique ones or so. When reflecting around the vertical direction, music speaks of an **'inversion'**. Reflection around the vertical direction (so in time) is called a **"retrograde"** or '**crab**' in music.	**A reflection in music** **Research question: A musician regularly uses a reflection in his piece of music. How can we recognize a reflection on a score?** The reflection was defined in the world of mathematics. We return to the world of music and study possible reflections in scores. **Assignment: Reflection in time (around vertical axis)** Listen to the piece of music below and watch the score. You can also study the accompanying line construction or look at the triangle. As with the translation/repetition, the musician has taken a motif and transformed it into a new piece of music by reflecting it. Can you discover the reflection? Draw the mirror axis on the score note below. 	Just as with the translation, it does not matter in mathematics which direction the mirror axis has, but in music it is a big difference whether you reflect over a horizontal (in the dimension of pitch) or vertical axis (reflection in time).



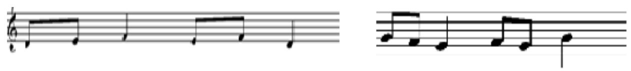
Solution:
In order to arrive at the new motif (the second piece of music), the musician used a reflection of the first piece of music around a **vertical axis**. In the world of music this is called a ‘**retrograde’** or **‘crab’**. Indeed a crab can walk backwards!

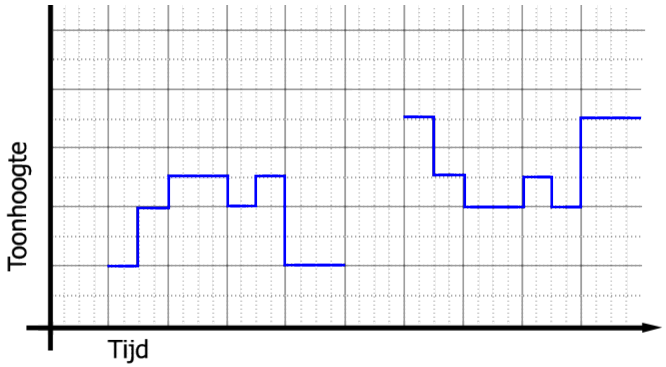
Assignment: Reflection in pitch (around horizontal axis)
Listen to the piece of music below and watch the score.
You can also study the accompanying line construction or look at the triangle.
Do you recognize the reflection of the motif?



Solution:
In order to arrive at the new motif (the second piece of music), the musician used a reflection of the first piece of music around a horizontal axis (reflection in pitch). In the world of music this is called an **inversion**.

Assignment:
Listen to the piece of music below and watch the score.
You can also study the accompanying line construction or look at the triangle.
Do you find how the motif was reflected?

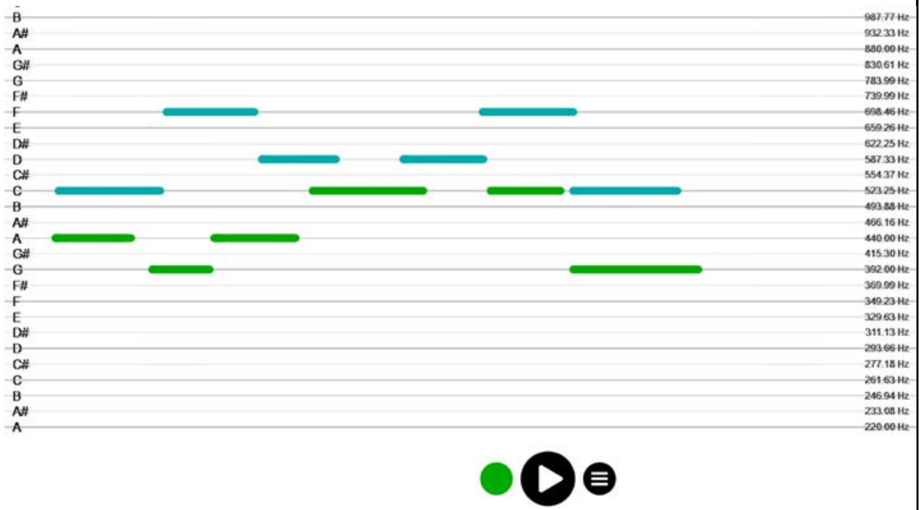
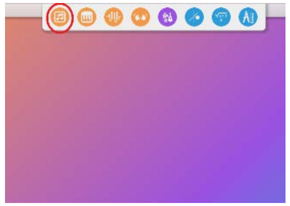


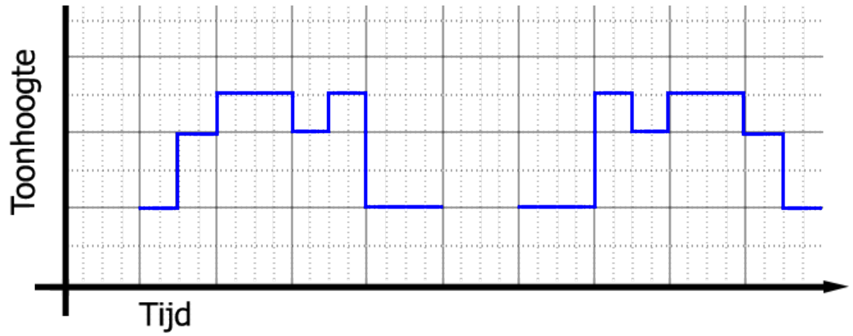


Solution:
In order to arrive at the new motif (the second piece of music), the musician first used a reflection of the first piece of music around a **horizontal axis** and then **translated it in (tone) height**.

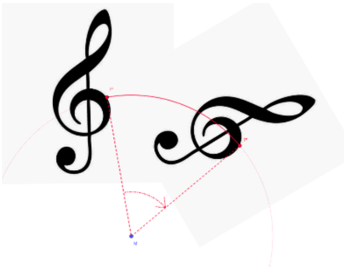
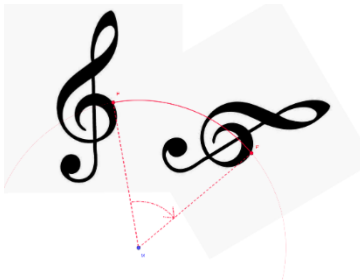
Assignment:
Which direction does the axis of mirrors have in the first command? (Vertical)
What is the direction of the mirror axis in the other commands? (- horizontal)
Is there any direction around which you could mirror in music?

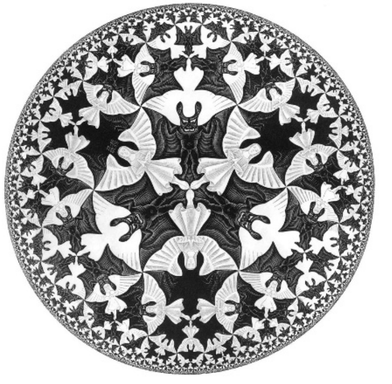
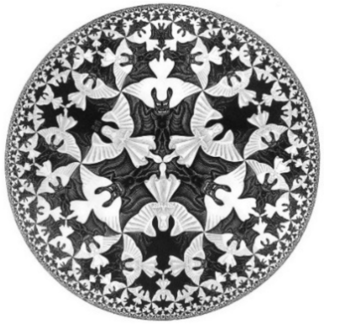
Conclusion:
When the music reflects around a vertical axis, we speak of a **temporal reflection** (the reflection takes place in time).
When the music is mirrored around a horizontal axis, we speak of a **tonal reflection** (the reflection takes place in the pitch).

	Create	M	Create your own piece of music with a reflection in it as motif processing.	**Assignment:** Create a piece of music yourself in the DrawMe tool in which a self-designed motif is mirrored. **Possible solutions:**  The blue motif shows the reflection of the first part of the motif around a vertical axis.	Use the DrAwMe-tool on the iMuSciCA workbench https://workbench.imuscica.eu/ 
	Engage /Imagine	A	We stay in the musical world and look further at the processing of motifs: the rotation	**A rotation in music** **Research question: A musician regularly uses a rotation in his music pieces. How can we recognize a rotation on a score?** **Assignment**: Listen to the piece of music below and watch the score. You can also study the line construction or look at the triangle. Do you recognize the motif processing in the piece of music?	

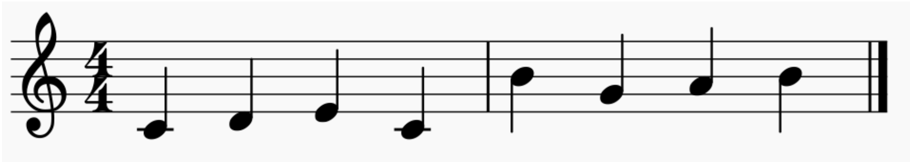
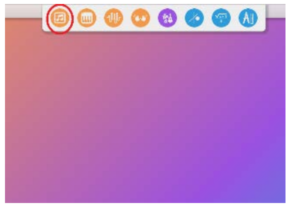
				   **Solution:** We see two pieces of music that show some similarity. The first piece of music is the same as the second piece of music, only played at 180°. The notes are, as it were, rotated 180° around a point in the score. We obtain the second motif by rotating the first motif 180° (in mathematics this is called reflection points). Starting from a motif, a musician can create an infinite number of other motifs by transforming them.	

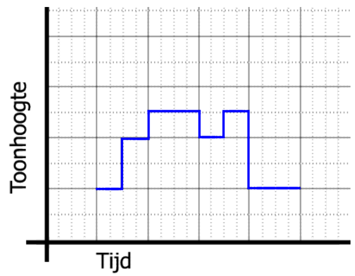
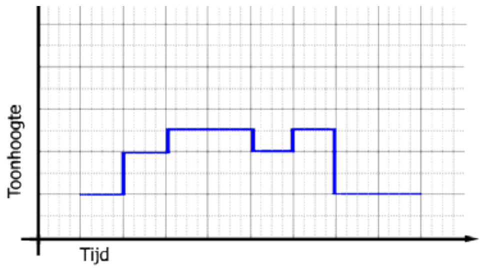
	Engage /Imagine	S	A rotation in mathematics should first be carried out by students using concrete material.	**A rotation in mathematics** **Research question: What is a rotation in mathematics? How can we describe a rotation mathematically?** **Assignment:** • Take a sheet of paper with the figure below on it. Place a transparent sheet on top and fasten the 2 sheets to each other with a split pin.  • Using a marker, mark the object on the transparent surface. • Now you can start rotating your transparent sheet, you can choose how: - Around which point do you turn the figure? (- the point where the split pin fixes the blades to each other) - How many different directions of rotation do you have? (-2 directions: clockwise and anti-clockwise) - Does the rotation of the figure over a quarter turn result in the same rotation as the rotation over 180°? (No)	**Concrete didactic material needed:** rotation pattern, transparent, split pin, marker
			Definition of elements that are necessary to perform a rotation mathematically.	**Definitions:** - The point around which we are turning is called the rotation point. The point P of the original figure is as far away from the pivot point as the rotational image P' of P. - The angle at which we rotate is called the rotation angle. - The rotation can be done in two directions. We speak of rotating clockwise or counter clockwise.	Definition of the mathematical terms: rotation point, rotation angle and direction.
	Investigate / Analyse	S	Performing a rotation using iMuSciCA's	**Assignment**:	Use the mathematics software Cabri.

			Geomtery and Algebra tools.	Draw an object of your choice or insert an object of your choice into the Geometry & Algebra tool on the workbench, place a pivot point and rotate the object around the pivot point 65° clockwise. Compare your result with the drawing below: 	Have a look at the document Have a look at the help document and discover [how geometical symmetry operations can be performed with iMuSciCA's Geometry & Algebra tools].
			The mathematical formulation of a rotation	**A rotation in mathematics** **Assignment:** Look at the figure below. Do you see the rotation of the clef around the rotation point O? Any point P of the original clef is rotated around point O to a point P' of the clef rotation pattern.  **Remark**: Note that the oriented angle PÔP' is equal to the angle of rotation in the direction of ration. **Assignment:**	We look here to the mathematical elements of a rotation.

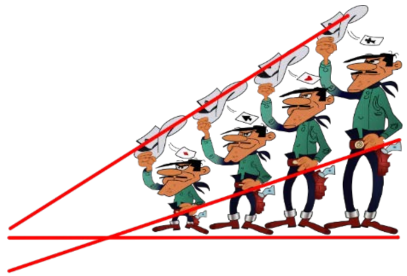
				Look at the figure below. What transformation can the artist Maurits Cornelis Escher have used to make the work of art? (- rotation or reflection) Compare your answer to that of your fellow students, have you found all possible transformations? 	
	Apply	W	Applying the definition of rotation	**Assignment:** Take the transparent and the sheet of paper on which the work of art below is printed and place it nicely together. Now find the pivot point in this figure yourself and place the split pin there. Did you manage to find the right place for the pivot point? By rotating, can you depict this figure on itself?  **Conclusion**:	The term rotational symmetry is being addressed here. **Material needed:** turning drawing on paper, turning drawing on transparent, split pen

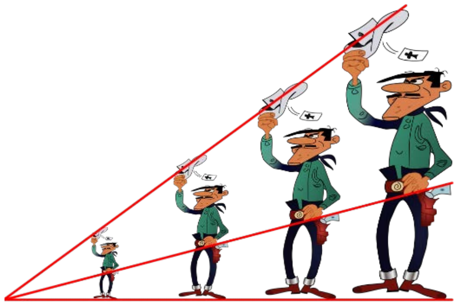
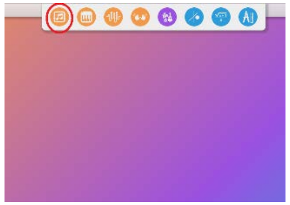
				A figure that is imaged by a rotation on itself is called a **rotational symmetrical object**. **Assignment:** Are the figures below rotationally symmetrical? (You may assume that they run infinitely in all directions). If yes, indicate a possible pivot point. (- the snowflake and flower come from nature and are therefore not quite perfectly symmetrical, but are a very good approximation). 	
	Apply/ Research	M	We return to music. What is learned from mathematics about rotation is applied. In musical pieces, you can only rotate over an angle of 180°. (i.e. point reflections in mathematics).	**A rotation in Music** **Research question: What are the rotation angles that can be used to rotate musical motives?** **Assignment:** Take a closer look at the music pieces below. Can you draw the pivot points? What are the rotation angles on which the musician rotate the motifs? Music piece 1:	Hint: In order to get a better understanding of the assignment with the students, the split pin can be used again: Use one long score with a short motif, attach the transparent with the split pin on the paper, making sure that the centre (the split pin) goes through the centre of the score. When the students are rotating, they see that the notes can only end up on the score when an angle of 180 ° has been used.

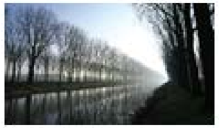
				 Music piece 2:  Music piece 3:  **Conclusion:** In music tracks, motifs can only be rotated 180°. (In mathematics this is also called a point reflection).	
	Create	M	Create your own piece of music with a turn in it as motif processing.	**Assignment:** Create your own piece of music in the DrAwMe tool, rotating a self-designed motif 180°. **Possible solution:** 	Use the DrAwMe-tool in the iMuSciCA workbencht https://workbench.imuscica.eu 

				The orange motif shows the rotation of the first part of the motif around a rotation angle of 180°.	
	Engage/Present	M	We stay in the musical world and look at the processing of motifs: homothecy(in music terms we speak of an augmentatio and diminutio).	**A homothecy in music** **Research question: A musician regularly uses a homothetic approach in his piece of music. How can we recognize a homothecy(an enlargement/reduction) on a score?** **Assignment**: Listen to the piece of music below and watch the score. You can also study the line construction or look at the triangle. Do you find the homothecy in the piece of music?    	

				 **Solution:** We see two pieces of music that show some similarity. The notes sound the same, but in the second piece of music the notes last longer. The length of the notes is, as it were, increased (doubled). The musician has enlarged the motif, processed it as an augmentation.	
	Engage/Present	S	Homothecies are taught in mathematics. Recognize an increase or decrease and know that this is called a homothecy.	**A homothecy in mathematics** **Research question: What is homothecy in mathematics? How can we describe a homothecy mathematically?** **Assignment**: What is noticeable when you look at the figures below, do you see a connection between the figures? (- These are always enlargements or reductions.)   Mark the tip of the tail and the noses of the mice. Draw the straight line through those different corresponding points. What is noticeable? (- these lines intersect in the same point.) **Conclusion:**	The mathematical term homothecy and centre are being addressed here.

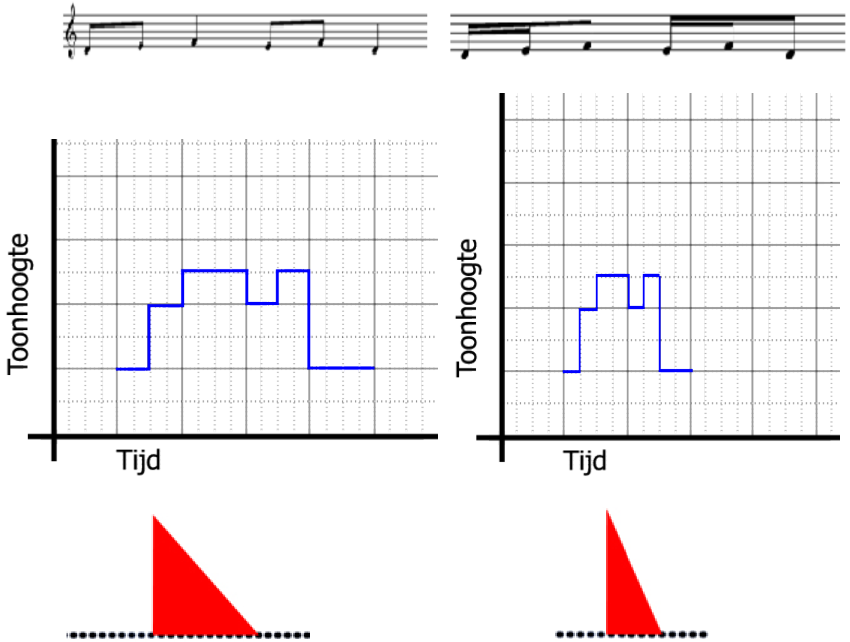
				A magnification or reduction is called a **homothecy** in mathematics. The intersection of the straight lines through corresponding points is called the **centre**.	
	Apply			**A homothecy in mathematics** **Assignment:** Look at the figure below. Can you also find the centre here? (No) Draw the lines through the corresponding points to check if they intersect in 1 point.  **Solution:** 	
	Investigate /Analyse	S	Performing a homothecy in mathematics with iMuSciCA's the Geometry & Algebra tool	**A homothecy in mathematics** **Assignment**: Insert the figure below into the Geometry & Algebra tool and place a selected point C that will serve as the centre.	Use the iMuSciCA's the Geometry & Algebra tool..

				In the Geometry & Algebra tool, perform the following homothecies from your chosen point C: use as reduction factors ¾, ½ and ¼. This way you will get the 'mathematically correct' version of the Dalton brothers. Draw the straight lines through the corresponding points to check whether they intersect in the centre C.  **Solution**: 	 Have a look at the help document and discover how geometical symmetry operations can be performed with iMuSciCA's Geometry & Algebra tools. (Remark: help for homothecy operations still in development)
			The mathematical formulation of a homothecy.	**Assignment:** Look at the figure below. Do you see the enlargement of the clef with centre C? Any point P of the original clef corresponds to a point P' of the enlarged image of the clef.	The mathematical theory of a homothecy with centre and vector. You can focus on this in more profound way in maths class.

				 **Observation**: Mathematically, |CP| = k |CP|, where C is the centre of the homothecy and k is the magnification factor. **Assignment:** Look again at the figure of the clef above. Which magnification factor was used? (k=2,5)	
	Apply	S	Applying the definition of a homothecy.	**Assignment:** Below you will find some more examples of homothecies. Can you find the centre in every image?    	

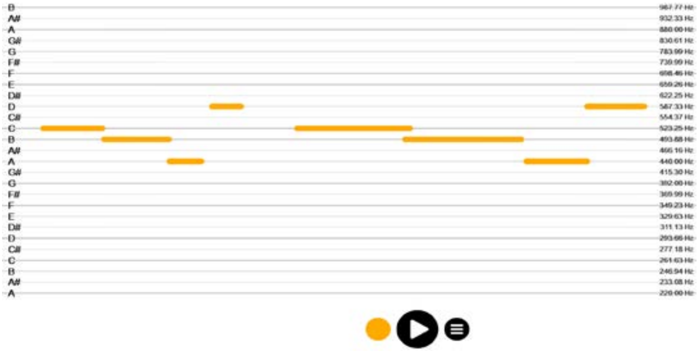
	Apply	A	We return to the music. It applies what was learned about homothecies from mathematics. Enlargements and reductions in music pieces are being studied.	**A homothecy in music** **Research question: How can a musician incorporate enlargements or reductions in music pieces?** A homothecy in music is a bit more difficult to compare with a homothecy in mathematics. The notes on the score remain drawn like balls of equal size, but their *length of play* can vary. One can make the duration of notes longer in time is (augmentation) or just shorter in time (diminution). **Assignment:** A composer or a musician could enlarge/reduce a motif in a piece of music in two different ways or dimensions. Which are they? **Solution**: There are indeed two possibilities: 1. The length of the notes can be enlarged or diminished (Augementation or Diminution in time) 2. The interval between the notes can be enlarged or diminished (Augementation or Diminution in pitch) The pitch can indeed be increased or decreased. In this case we prefer to say that the musical interval between the notes increase / decrease. For example: consider the musical interval between a G (sol) and an A (la) consists of 1 tone. The musical interval between a sol and a si consists of 2 tones. So the pitch is doubled or multiplied by 2. **Assignment:** Examine the 4 pieces of music below. In each piece of music, the iMuSciCA motif is transformed in a different way. For each piece of music, check whether it is an enlargement/reduction, or whether it is an enlargement/reduction in the playing length of the notes or in the distances between notes. Also specify which factor will increase or decrease, e.g. doubling the musical interval. Musical piece 1:	

Musical piece 2:



Musical piece 3:

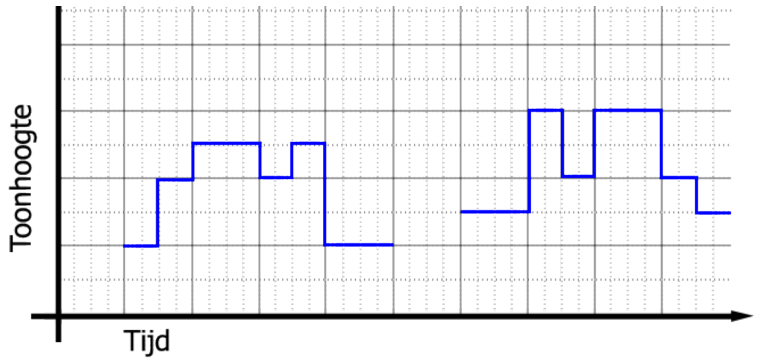
Musical piece 4:

				Solution: 1. enlargement of the length of the notes by a factor of 2. This is called an augmentation ratio in music. 2. reduction of the length of the notes by a factor of 1/2. This is called a diminutio in music. 3. Enlargement of the musical interval by a factor of 3. 4. Reduction in musical interval by a factor of 1/2.	
			Create your own piece of music using an augmentation ratio or diminutio as motif processing.	**Assignment:** Create a piece of music yourself in the DrAwMe tool that enlarges or reduces a self-designed motif. **Possible solution:**  In the orange motif, the second part has become twice as long as the first part of the motif. This is an augmentation ratio.	Use the DrAwMe-tool in the iMuSciCA workbencht https://workbench.imuscica.eu
	Communicate/ Reflect	A / S	What did you find out? Discuss and come to conclusions.	Show what you have discovered so far about the following symmetry operations/ transformations/ motif processing: 1. The translation 2. The reflection 3. The rotation 4. The homothecy	The pupils can, for example, search for the transformations in their own environment and in the pieces of music they know.

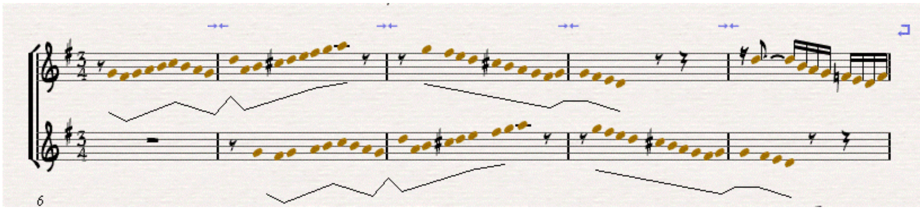
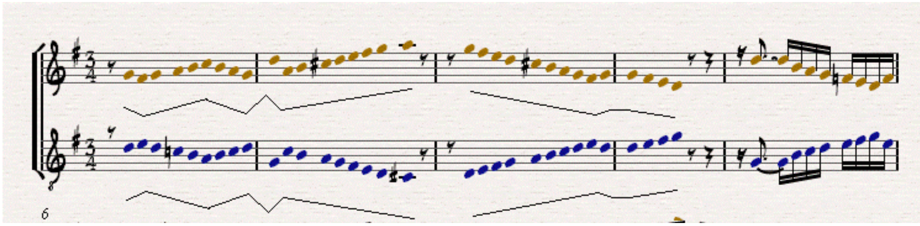
				The class is divided into 4 groups and each group has to propose 1 transformation/motif processing to the class (preferably in an original way). Both the transformation in mathematics and the motif processing in music need to be addressed.	
				"Well done, you've now got to know the different transformations, both in the world of music and in the world of mathematics!”	

Lesson plan 4.3: **Combinations of transformations in music and mathematics**

Time	Phases	Field	Description	Activity	Remarks
1	Apply	A / S	We look at combinations of motif processing in more complex pieces of music and recognize the mathematical transformations in them.	**Research question: When studying more complex pieces of music, can we recognize the different transformations/motifs that are used simultaneously and not one after the other as in the previous lesson plan?** **Assignment:** Listen to the piece of music below and watch the score. You can also study the line construction or look at the triangle. Do you find transformations of motifs in the piece of music and what are they? 	Combinations of transformations from lesson 4.2 are made, this time immediately viewed from both domains, music and mathematics.

				  **Solution**: In music: Incorporation of motives by reflection/inversion in time, then transposed/ translated in pitch. (This is called an inverse melodically varied sequence in the music). In mathematics: a reflection around a vertical axis followed by a translation in vertical upwards direction.	
				Assignment: Listen to the piece of music below and watch the score.	When the scenario is addressed in music class, or at least in presence of a music teacher, these assignments can be made. In other cases they might be skipped.

				Mark the motif. Do you see the motif returning, slightly differently processed by the musician? Name the motif transformation. 	
				Assignement: Listen to the piece of music below and study the score carefully (Symmetry in the Goldberg Variations, Goldberg Variations BWV 988 Var 12 - Canone alla Quarta, composer: Johann Sebastian Bach (1685-1750), performance: Glenn Gould). Which motif transformations can you discover? 	

				Solution: Translation in time:  Reflection in pitch: 	
				“Congratulations, you have now experienced the different connections between the world of music and the world of mathematics!”	